NANYANG JUNIOR COLLEGE

DEPARTMENT OF MATHEMATICS

MATHEMATICS

Year One (2008)
Tutorial 2A: Binomial Expansion

1. Expand the following as a series of ascending powers of x, giving the first 3 terms and the term in x^n. State for what set of values of x the expansions are valid.

   (a) $\dfrac{-3}{x+1}+\dfrac{4}{x+4}$ [$-2+\dfrac{11}{4}x-\dfrac{47}{16}x^2; |x|<1; \left(3(-1)^{n+1}+(-1)^n\left(\dfrac{1}{4}\right)\right)x^n$]

   (b) $\dfrac{1}{(2-x)^2}$ [$\dfrac{1}{4}+\dfrac{1}{4}x+\dfrac{3}{16}x^2; |x|<2; \dfrac{(n+1)}{4}\left(\dfrac{x}{2}\right)^n$]

2. Find the first three terms in the series expansion of $\left(4+\dfrac{1}{x^2}\right)^{-1}$, in

   (a) **ascending** powers of x. [$x^2-4x^4+16x^6; -\dfrac{1}{2}<x<\dfrac{1}{2}$]

   (b) **descending** powers of x. [$\dfrac{1}{4}-\dfrac{1}{16}x^{-2}+\dfrac{1}{64}x^{-4}$; $x<-\dfrac{1}{2}$ or $x>\dfrac{1}{2}$]

   In each of the above cases, state the set of values of x for which the expansion is valid.

3. Show that the expansion of $(1-x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 is $1-\dfrac{1}{2}x-\dfrac{1}{8}x^2-\dfrac{1}{16}x^3$. State the set of values of x for which the expansion is valid. By taking $x=\dfrac{1}{64}$, use this expansion to evaluate $\sqrt{7}$, correct to 5 decimal places. [$-1<x<1$; 2.64575]

4. Express $\dfrac{2x+3}{(1+x^2)(2-3x)}$ in partial fractions. [$\dfrac{3}{2-3x}+\dfrac{x}{1+x^2}$]

   Given that x is sufficiently small for x^3 and higher powers of x to be neglected,

   show that $\dfrac{2x+3}{(1+x^2)(2-3x)}=\dfrac{3}{2}+\dfrac{13}{4}x+\dfrac{27}{8}x^2$.

5. Express f(x) = $\dfrac{16-9x}{(1-3x)(4+x)}$ in partial fractions and hence, obtain f(x) as a series in ascending powers of x, giving the first 3 non-zero terms of this expansion. State the set of values of x for which the expansion is valid. Find also the term in x^n in the expansion.

$[\dfrac{3}{1-3x}+\dfrac{4}{4+x};\ 4+\dfrac{35}{4}x+\dfrac{433}{16}x^2;\ -\dfrac{1}{3}<x<\dfrac{1}{3}]$

6. [HCI 2006 JC1 MYE/Q8]

Given that $\left(1+\dfrac{x}{a}\right)^m = 1-\dfrac{x}{4}+\dfrac{3x^2}{32}+...$, where $m<0$,

(i) Find the values of a and m . Hence state the range of values of x for which the binomial series is valid. $[a=2; m=-\dfrac{1}{2}; |x|<2]$

(ii) Verify that the term in x^3 has coefficient $-\dfrac{5}{128}$.

(iii) Use your expansion up to and including the term in x^3 to calculate the value of $\sqrt{17}$ to three significant figures. [4.12]

1. Assignment

Show that $\left(\dfrac{4x+1}{x}\right)^{-\frac{1}{2}} = \dfrac{1}{2}\left(1+\dfrac{1}{4x}\right)^{-\frac{1}{2}}$.

Find the expansion of $\left(\dfrac{4x+1}{x}\right)^{-\frac{1}{2}}$ in ascending powers of $\dfrac{1}{x}$ up to and including the term in $\dfrac{1}{x^2}$. State the set of values of x for which this expansion is valid. Hence, by substituting a suitable value of x in the expansion, find an approximation for $\left(\dfrac{40}{41}\right)^{\frac{1}{2}}$, giving your answer correct to 5 decimal places.