NANYANG JUNIOR COLLEGE

DEPARTMENT OF MATHEMATICS

H2 Mathematics 9740 **Tutorial 02: Binomial Theorem** **Year 1/2008**

1. Find the first four terms in the expansions of the following:

   (a) $(1+2x)^8$ (b) $\left(4-\frac{1}{7x}\right)^6$ (c) $\left(2x-\frac{1}{2x^3}\right)^8$

   [$1+16x+112x^2+448x^3$, $4096-\frac{6144}{7x}+\frac{3840}{49x^2}-\frac{1280}{343x^3}$, $256x^8-512x^4-224x^{-4}+70x^{-8}$]

2. Find the indicated term in each of the following expansions.

   (a) $(3x-2)^{10}$, 6^{th} term (b) $\left(2x-\frac{1}{x}\right)^9$, coefficient of x^7

   [$-1959552x^5$, -2304]

3. Express $(x+2)^5-(x-2)^5$ as a polynomial in x and hence find the exact value of $\left(\sqrt{5}+2\right)^5-\left(\sqrt{5}-2\right)^5$. Use the fact that $0<\sqrt{5}-2<\frac{1}{4}$ to deduce that $\left(\sqrt{5}+2\right)^5$ differs from an integer by less than $\frac{1}{1024}$.

   [$20x^4+160x^2+64$; 1364]

4. Expand $(1+\frac{1}{x})^{-1}$

   (i) as a series in ascending powers of x up to and including the term in x^3.

   (ii) as a series in ascending powers of $1/x$ up to and including the term in $1/x^3$.

   Give the range of values of x for which each expansion is valid.

   [(i) $x-x^2+x^3+\ldots, |x|<1$;(ii) $1-\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^3}+\ldots, |x|>1$]

5. Expand the following as a series of ascending powers of x, up to and including the terms in x^2, expressing the coefficients in their simplest form. State for what set of values of x the expansions are valid: (a) $(8+5x)^{2/3}$ (b) $\frac{2-3x}{\sqrt{1+x}}$.

   [(a) $4+\frac{5x}{3}-\frac{25x^2}{144}, |x|<\frac{8}{5}$;(b) $2-4x+\frac{9}{4}x^2$; $|x|<1$]

6. Find the coefficient of x^6 in the expansion in ascending powers of x of

   (a) $\left(1+4x^2\right)^{1/2}$ (b) $\frac{1}{(1+2x^2)^2}$.

   [4,-32]

7. Expand the following as a series of ascending powers of x, giving the first 3 terms and the term in x^n. State for what set of values of x the expansions are valid.

   (a) $\frac{-2}{x+1}+\frac{3}{x+3}$ [$-1+\frac{5}{3}x-\frac{17}{9}x^2$; $\left[2(-1)^{n+1}+\left(-\frac{1}{3}\right)^n\right]x^n$; $|x|<1$]

   (b) $\frac{1}{(4-x)^2}$ [$\frac{1}{16}+\frac{x}{32}+\frac{3x^2}{256}$; $\frac{(n+1)x^n}{2^{2n+4}}$; $|x|<4$]

   (c) $(2-x)^{-3}$ [$\frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2$; $\frac{(n+1)(n+2)}{2^{n+4}}$; $|x|<2$]

8. (i) Express $f(x) = \dfrac{3x+5}{(x+1)(x+2)(x+3)}$ in partial fractions.

(ii) Given that x is sufficiently small for x^3 and higher powers of x to be neglected, show that

$f(x) \approx \dfrac{5}{6} - \dfrac{37}{36}x + \dfrac{227}{216}x^2$. [$\dfrac{1}{x+1} + \dfrac{1}{x+2} - \dfrac{2}{x+3}$]

9. [N91/1/14]

Let $f(x) = \dfrac{x^2+5x}{(1+x)(1-x)^2}$ Express f(x) in partial fractions. The expression of f(x), in ascending powers of x, is $c_0 + c_1x + c_2x^2 + c_3x^3 + \ldots + c_rx^r + \ldots$ Find c_0, c_1, c_2, and show that c_3=11.

Express c_r in terms of r. [$-\dfrac{1}{1+x} - \dfrac{2}{1-x} + \dfrac{3}{(1-x)^2}$ 0, 5, 6, $c_r = 3r + 1 - (-1)^r$]

10. (i) Express $\dfrac{5}{(x^2+1)(x-2)}$ in partial fractions.

(ii) Given that $|x| < 1$, expand $\dfrac{5}{(x^2+1)(x-2)}$ in ascending powers of x, up to and including the term in x^2. [$\dfrac{-x-2}{x^2+1} + \dfrac{1}{x-2}$; $-\dfrac{5}{2} - \dfrac{5x}{4} + \dfrac{15x^2}{8}$]

11. Find the expansion in ascending powers of x, up to and including the term in x^3, of $\dfrac{\sqrt{4-x}}{2x^2-1}$, simplifying the coefficients. State the range of x for which the expansion is valid.

[$-2 + \dfrac{1}{4}x - \dfrac{255}{64}x^2 + \dfrac{257}{512}x^3$; $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$]

12. [AJC 2005 Prelim/1/3]

Find the series expansion, in ascending powers of x, of $\dfrac{x+1}{\sqrt{1-2x}}$ up to and including the term in x^3. State the values of x for which this expansion is valid.

Hence, by putting $x = \dfrac{1}{11}$ in your expression, estimate $\sqrt{11}$ to 3 significant figures.

[$1 + 2x + \dfrac{5}{2}x^2 + 4x^3$; $|x| < \dfrac{1}{2}$; 3.32]

13. Find the expansion in ascending powers of x, up to and including the term in x^2, of $\left(\dfrac{1-x}{1+x}\right)^{\frac{1}{3}}$ where $|x| < 1$, simplifying the coefficients. By putting x to a suitable value, use your series to find an approximate value for $\left(\dfrac{15}{17}\right)^{\frac{1}{3}}$, giving your answer in the form of a fraction.

[$1 - \dfrac{2}{3}x + \dfrac{2}{9}x^2$; $\dfrac{1105}{1152}$]

14. [RJC 2005 Prelim/1/5]

Expand $(4+y)^{\frac{1}{2}}$ in ascending powers of y up to and including the term in y^3. Simplify the coefficients.

In the expansion of $(4+8x+kx^2)^{\frac{1}{2}}$, where k is a constant, the coefficient of x^3 is zero. By writing $(8x+kx^2)$ as y, find the value of k. [$2 + \dfrac{y}{4} - \dfrac{y^2}{64} + \dfrac{y^3}{512}$; $k = 4$]