



# NANYANG JUNIOR COLLEGE

## DEPARTMENT OF MATHEMATICS

JC 1 (2008)

H2 Mathematics

Tut 3 : Inequalities & Systems of Linear Equations

1. Solve the following inequalities:

a)  $x^2 - 5x + 4 \leq 0$

b)  $6 - x - x^2 > 0$

c)  $x(x - 2) < 5$

d)  $3x(x - 5) < 2(2x - 3)$

e)  $(1 + x)^2 > 2(1 + x)$

f)  $(x + 3)(2x - 1)(2 - x) < 0$

g)  $(x^2 + 1)(x^2 + x - 2) < 0$

h)  $x^2(x - 1)(1 - 4x^2) \leq 0$

2. Solve the following inequalities:

a)  $\frac{2x - 3}{x - 5} > \frac{3}{2}$

b)  $\frac{12}{x - 3} < x + 1, x \neq 3$

c)  $1 < \frac{4x - 4}{2x + 3} < 3$

d)  $\frac{x - 1}{2x - 1} < \frac{x}{4 - 3x}$

3. Find the solution set of the following inequalities:

a)  $|3x + 5| < 4$

b)  $|2x - 4| \geq 7$

c)  $|x - 2| < 2x$

d)  $|3x + 2| \geq 1 - x$

e)  $|3 - 2x| \leq |x + 4|$

f)  $\left| \frac{x - 5}{1 - x} \right| > 1$

g)  $\frac{2|x| - 1}{x^2 + 1} > \frac{1}{3}$

h)  $|x| \leq \frac{2}{|x| - 1}$

4. N92/1/4

Sketch, on a single diagram, the graphs of  $x + 2y = 6$  and  $y = |x + 2|$ . Hence, or otherwise, solve the inequality  $|x + 2| < \frac{1}{2}(6 - x)$ .

5. Sketch the graph of  $y = |3 - 2x| - |x + 4|$  and hence solve  $y \leq 0$ .

6) Let  $f(x) = \frac{x(x - 1)}{x + 2}$ ,  $x \neq -2$ . Solve the following inequalities:

a)  $f(x) < 2$

b)  $f(|x|) < 2$

7. By considering the sketch of  $y = |2x + 1|$  and  $y = 1 - x^2$  on the same diagram, solve the inequality  $|2x + 1| - 1 + x^2 > 0$ .

8. Show that, if  $x$  is real,  $2x^2 + 6x + 9$  is always positive. Hence, or otherwise, solve the inequality  $\frac{(x+1)(x+3)}{x} > \frac{x+6}{3}$ .

9. Find the range of values of  $x$  which satisfy the inequality  $\frac{1}{x-1} > \frac{2}{x+2}$ . Hence

deduce the values for  $\frac{1}{1-e^x} > \frac{2}{1+2e^x}$ .

10. Find the quadratic function,  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are constants, whose graph passes through the points  $(1, -5)$  and  $(-1, 1)$  and cuts the  $y$ -axis at  $-4$ .

11. A company makes three types of toy bears: Papa bear, Mama Bear and Baby Bear. The amount of materials required (in units) for each type of bear is given below.

	Plastic	Rubber	Cloth
Papa Bear	2	1	1
Mama Bear	3	4	5
Baby Bear	1	2	2
Total amount available	140	200	210

If all the material available are used, how many bears of each type does the company make?

### Answers

- 1a)  $1 \leq x \leq 4$       b)  $-3 < x < 2$       c)  $1 - \sqrt{6} < x < 1 + \sqrt{6}$   
d)  $1/3 < x < 6$       e)  $x < -1$  or  $x > 1$       f)  $-3 < x < 1/2$  or  $x > 2$   
g)  $-2 < x < 1$       j)  $-1/2 \leq x \leq 1/2$  or  $x \geq 1$   
2a)  $x < -9$  or  $x > 5$       b)  $-3 < x < 3$  or  $x > 5$       c)  $x < -13/2$  or  $x > 7/2$   
d)  $1/2 < x < 4/3$   
3a)  $-3 < x < -1/3$       b)  $x \leq -3/2$  or  $x \geq 11/2$       c)  $x > 2/3$   
d)  $x \leq -3/2$  or  $x \geq -1/4$       e)  $-1/3 \leq x \leq 7$       f)  $x < 3$ ,  $x \neq 1$   
g)  $-3 - \sqrt{5} < x < -3 + \sqrt{5}$  or  $3 - \sqrt{5} < x < 3 + \sqrt{5}$       h)  $-2 \leq x < -1$  or  $1 < x \leq 2$   
4)  $-10 < x < 2/3$       5)  $-1/3 \leq x \leq 7$   
6a)  $x < -2$  or  $-1 < x < 4$       b)  $-4 < x < 4$       7)  $x > 0$  or  $x < 1 - \sqrt{3}$       8)  $x > 0$   
9)  $x < -2$  or  $1 < x < 4$ ;  $-\ln 4 < x < 0$   
10) 2, -3, -4      11) 20, 10, 70